Active Control of Configuration-Dependent Linkage Vibration with Application to a Planar Parallel Platform

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Abstract—A new lightweight planar parallel platform aims to greatly improve operational speed of electronic manufacturing process and to realize a “smart parallel platform” through the integration of a parallel mechanism architecture, new sensing techniques and active vibration control using Lead Zirconate Titanate (PZT) transducers. Experiments using an accelerometer, an impact hammer, and PZT transducers are performed to accomplish Experimental Modal Analysis (EMA), which has not been thoroughly investigated on flexible mechanisms. Through EMA, vibration characteristics of flexible linkages, when the platform is stationary and in motion, are modeled, with configuration-dependent vibration observed. Based on assumptions made in EMA, a modal control strategy using constant mode shapes is designed to control configuration-dependent linkage vibration. Active vibration control experimental results, showing a 40%-50% reduction of vibration amplitudes, validate the effectiveness of the control strategy. This control strategy will be generalized to achieve configuration-dependent vibration control, hence realizing a true smart parallel platform.

Index Terms—active vibration control; experimental modal analysis; modal control; parallel manipulator; smart structures;

I. INTRODUCTION

Suppression of unwanted vibration of in high-speed lightweight mechanisms caused by flexible linkages has attracted tremendous effort from many researchers. A variety of techniques, including topology optimization of linkage cross-section, utilization of high-performance composite materials, and passive damping materials, have been proposed to reduce the linkage vibration response during high-speed motion in order to achieve high-precision motions. Recent emergence and development of smart structure techniques [1] provide a new solution for vibration suppression “Smart structures” adopt microprocessors and distributed actuators and sensors to dynamically modify structural dynamics to actively suppress vibration in their working environments. Different from previous approaches, smart structure control approach introduces “adaptability” or “intelligence” to the controlled mechanism, which is especially desirable for flexible mechanisms where vibration characteristics are rather complex, exhibiting configuration-dependency and high nonlinearity. A “smart parallel platform”, shown in Fig. 1, featuring the integration of a parallel mechanism architecture, new sensing techniques, data fusion and active vibration control of linkage vibration using PZT components, is proposed and analytically modeled using a substructuring approach [2]. This paper addresses experimental modal analysis and active vibration control design of flexible linkage in the smart parallel platform.

Pioneering attempts [3]-[8] have been undertaken to introduce Lead Zirconate Titanate (PZT) material, which is a common smart material, to active vibration control of flexible mechanisms. Based on an analytical constant modal model, Sung et al. [3]-[4] numerically and experimentally investigated the usage of one PZT sensor and one PZT actuator, i.e. a single-input single-output (SISO), to suppress vibration of a four-bar linkage mechanism. Choi et al. [5]-[8] simulated vibration control of a flexible mechanism using piezoelectric films using an Assumed Mode Method (AMM). We note that the active vibration control research work on flexible mechanisms cited above is either numerical simulation or SISO control experiment. A single sensor is insufficient to deal with configuration-dependent active vibration control. In order to realize smart structures on mechanisms, distributed PZT transducers must be utilized. Experimental identification and active vibration control using distributed PZT transducers is essential to the realization of true smart structures in flexible mechanisms.

Experimental Modal Analysis (EMA) [9] techniques have been widely used in the experimental identification of structural dynamic characteristics. Studies on experimental identification of flexible mechanisms are few compared with studies on numerical simulation of flexible multi-body systems [10], usually accompanying with configuration-dependency and nonlinearity. The AMM or Finite Element Method (FEM), which often gives different results from experimental results, is usually adopted when linkage vibration is considered. In his review, Shabana [10] pointed out that development of experimental modal analysis
The configuration of the parallel platform is $R-R=\rho_1 \rho_2 \rho_3$ with respect to the location of the sliders. The origin of the coordinate system is selected respectively. Three active prismatic joints, i.e. the three sliders shown in Fig. 2, are jointed to the moving platform with a revolute joint and two revolute joints in sequence. The three kinematic chains is respectively comprised of a prismatic joint and two revolute joints in sequence. The three prismatic joints, i.e. the three sliders shown in Fig. 2, are active joints, at $B_i, i=1,2,3$. Each prismatic joint is driven by a DC brushless motor AeroTech BM200. Assembled with this motor using a ball screw linear guide system THK KR3306CS00LP, each slider comprises a prismatic joint. Each slider is connected to a 200mm-long flexible linkage through a revolute joint at $B_i, i=1,2,3$. The flexible linkage is jointed to the moving platform with a revolute joint at $C_i, i=1,2,3$. Both revolute joints at $B_i$ and $C_i$ are passive joints. Through actuation of the three DC motors, the moving platform with a side length of 100mm and a mass of 4.049kg, is driven to a desired location in the plane within the workspace, approximately 40cm x 40cm, with a desired orientation. Four PZT sensors and two PZT actuators for active vibration control are bonded to each flexible linkage.

In Fig. 2, a global coordinate system is shown as $(x, y, \phi)$. The origin of the coordinate system is selected at the center of the workspace, $O$. The position and orientation of the platform, at the mass center $P$, is defined as $(x_p, y_p)$ and $\phi_p$ respectively. Three active prismatic joints, i.e. sliders, moves along ball screws, whose orientations are at angles $\alpha_i$ with respect to the $x$ axis, respectively 150°, 270°, 30°. The origins of three ball screws are indicated by $A_i, i=1,2,3$. The coordinate from $A_i$ to the location of the sliders $B_i$ is defined as $\rho_{i}, i=1,2,3$. The configuration of the parallel platform is denoted by $\{\rho_1 \rho_2 \rho_3\}$ with units of mm.

The experimental system control system is comprised of the parallel platform, three sets of motion control instruments, active vibration control instruments and a host PC with a MCX-DSP controller, which features 120 mega floating-point operations per second to simultaneously perform motion and vibration control. Motion control driving signals are conditioned by BAS320 amplifiers, which converts voltage signals form the motion controller into current signal so that motors work in torque (current) mode.

Fig. 3 shows a top view of the active vibration control system for one linkage. The signal from a PZT sensor is a charge signal with high impedance. Before it can be acquired by the DSP controller, it should be converted to a higher-level voltage signal with low impedance. An in-line impedance converters, Kistler 558, are adopted to do so. The converted voltage signal then is amplified to a desired level by a Kistler coupler 5134. These amplified signals with proper voltage ranges then are properly captured by the DSP controller. The DSP controller also provides PZT drive signals which are filtered by second order filters for the active control law. The DSP controller then sends driving signals to the PZT actuators to control vibrations of the platform.

The paper is organized as follows. Section II introduces the working principle, the coordinate system of the parallel platform, and the active vibration control experimental setup. Section III describes the procedure and results of the EMA of a flexible linkage in the parallel platform. Vibration of the linkage is characterized through EMA. Based assumptions made using EMA results, Section IV designs the active vibration control law. Experimental results of active vibration control are also given. Section V concludes the paper.

II. EXPERIMENTAL SETUP

Fig. 2 shows a schematic of the proposed smart planar parallel platform, whose architecture is categorized as a P-R-R (Prismatic-Revolute-Revolute) type, i.e. each of three kinematic chains is respectively comprised of a prismatic joint and two revolute joints in sequence. The three prismatic joints, i.e. the three sliders shown in Fig. 2, are active joints, at $B_i, i=1,2,3$. Each prismatic joint is driven by a DC brushless motor AeroTech BM200. Assembled with this motor using a ball screw linear guide system THK KR3306CS00LP, each slider comprises a prismatic joint. Each slider is connected to a 200mm-long flexible linkage through a revolute joint at $B_i, i=1,2,3$. The flexible linkage is jointed to the moving platform with a revolute joint at $C_i, i=1,2,3$. Both revolute joints at $B_i$ and $C_i$ are passive joints. Through actuation of the three DC motors, the moving platform with a side length of 100mm and a
Butterworth low pass filters and conditioned by a SA-21 signal amplifier, powered by a SA-10 power supply. An HP35670A dynamic signal analyzer, programmable by the host PC, is used to perform signal analysis and data logging.

III. EXPERIMENTAL IDENTIFICATION OF A FLEXIBLE LINKAGE

This section investigates the experimental identification of flexible linkages of the smart parallel platform. The configuration of one flexible linkage is illustrated in Fig. 3. There are four PZT sensors, regarded as point sensors due to their limited sizes in the beam axial direction. A miniaturized accelerometer PCB A352C67 is sequentially placed at different locations, ranging from point 1 to point 19 in Fig. 3. In the following, Subsection A summarizes the modal analysis procedure introduced in [14]. Subsection B presents experimental results of modal identification of the flexible linkages. Subsection C analyzes the influence of rigid body motion on the linkage flexible motion, taking axis 1 as the input of rigid body motion. Configuration dependency of the influence of rigid-body motion on flexible motion and linkage mode properties is then discussed in detail in Subsection D.

A Procedure of experimental modal analysis

Point PZT sensors, shown in Fig. 3, sense strain signals at the midpoints of PZT sensors, i.e. at point 4, 7, 13, and 16. PZT actuators generate a pair of bending moments at two ends of PZT actuators, coincident with locations of PZT sensors. Strain mode shapes measured by PZT transducers are required during real-time control of the smart parallel platform. Due to the fact that the number of PZT sensors is limited to give a clear insight of the mode shapes, accelerometers are also used in the modal tests since the accelerometer is readily placed at different locations on the linkage. In order to accurately evaluate the modal influence factors of PZT actuators, a standard impact hammer is used for the comparison. In [14], five types of frequency response functions (FRF) are defined: (1) Type I: an impact hammer as the excitation device and accelerometers as pick-up devices, which combination is conventionally used; (2) Type II: An impact hammer as the excitation device and PZT sensors as pick-up devices; (3) Type III: A bending moment as excitation and accelerometers as pick-up devices; (4) Type IV: A PZT actuator as the excitation device and accelerometers as pick-up devices; (5) Type V: A PZT actuator as the excitation device and PZT sensors as pick-up devices.

Based on a Global Rational Fraction Polynomial (GRFP) method [15], Wang and Mills [14] proposed a modal analysis procedure to identify modal shapes by using the above summarized transducers combinations. This procedure fits measured FRF curves using Forsythe orthogonal polynomials [16], then converts those orthogonal polynomial coefficients obtained into rational

![Fig. 3. Configuration of vibration control system for one flexible linkage and allocation of sensors](image)

![Fig. 4. Type I FRFs of the flexible linkage](image)

![Fig. 5. First two mode shapes identified using type I FRFs](image)

![Fig. 6 Type V FRFs of the flexible linkage](image)

![Fig. 7. The first two strain mode shapes of the flexible linkage](image)
polynomial coefficients. Based on rational polynomials, modal parameters are identified, including modal frequencies, modal shapes, and damping ratios. This procedure has been coded in the Matlab environment.

### B Identification of linkage mode shape

Intensive modal identification experiments have been performed on linkage 1 shown in Fig. 3. In this subsection, the flexible linkage is kept stationary and no overall rigid body motion is involved.

Type I FRFs defined in subsection above are used to identify the mode shapes using following the procedure in [14]. The accelerometer is placed sequentially from point 1 to point 19. Fig. 4 shows these nineteen Type I FRFs when the impact hammer hits the linkage point 18 when the planar parallel platform is at the home configuration. Using these FRFs, the first two mode shapes are identified, shown in Fig. 5. The first natural frequency is 94.14Hz with damping ratio of 2.4%. The second natural frequency is 323.71Hz with damping ratio of 1.07%. Type IV FRFs, using PZT actuators and an accelerometer, also give consistent results with the results of Fig. 5, which is not shown here due space limitations.

Type V FRFs, using the combination of PZT actuators and PZT sensors, are acquired in five different configurations to investigate the strain mode shapes and their configuration dependency. Six configurations \{ρ₁, ρ₂, ρ₃\}, are respectively selected as: \{0 0 0\}, \{30 0 0\}, \{60 0 0\}, \{-30 0 0\}, \{-50 0 0\}, and \{-50 60 10\}. Fig. 6 shows Type V FRFs when PZT actuator 1 is used to excite the flexible linkage when the parallel platform is at the configuration 1, i.e. the home configuration. The analysis of FRFs of this type gives the normalized mode shapes at different configurations as in Fig. 7 and the mode influence factors of PZT actuators. The mode shapes, given by PZT sensors, are in the terms of strain. Through analysis, it is found that strain mode shapes are approximately proportional to the second order derivative of the displacement mode shapes in Fig. 5. From Fig. 7, it is observed that the linkage mode shapes are kept while the natural frequencies are found with slightly shifts. The difference of the first natural frequency in Configuration 1 and Configuration 5 is found to be 13.09Hz. Because over all rigid body motion is not considered, vibration is local to the linkage. Hence, modal characteristics are supposedly linear and time-invariant except near singularity configurations. The variation is attributed to the variation in the constraints at the linkage boundaries, which basically remain pinned-pinned except singularity configurations. Experimental results validate this assumption.

### C Flexible motion caused by rigid body motion

In order to model the influence of rigid body motion on linkage flexible motion, frequency response functions from motor voltage input to linkage vibration is also measured and analyzed. Here motor 1 input is chosen as the rigid body motion input. During the measurement, motor 1 is set to have a short-distance movement of 5mm over 25ms so that the vibration characteristics are considered constant during the motion. Thus, the experimental modal analysis procedure in [14] is applicable. In order to understand the vibration characteristics more thoroughly, acceleration measurement is used where the accelerometer is sequentially placed at 19 points shown in Fig. 3. Fig. 9 shows 19 FRFs when the configuration of the platform is moved from \{0 0 0\} to \{5 0 0\} with a setpoint speed of 200 mm/s and setpoint acceleration of 25 g. It is observed that frequency components are much more complicated than those in Fig. 4 and Fig. 6, where the vibration is local to the flexible linkage. In Fig. 8, vibration of the entire parallel platform is involved, including flexible motion of linkages and other transmission components and overall rigid body motion. Fig. 9 gives the first six major mode shapes obtained from the analysis. Some peaks close to each other give very similar mode shapes, which are not plotted here. It is also concluded that the mode shapes at 101Hz and 323Hz are nearly identical to those two identified in Fig. 6.

The configuration dependency of the influence of rigid body motion on flexible motion is also investigated. Fig. 10 shows linear spectrums of the acceleration at point 18 when a command of 5 mm translation is sent to the motor 1 starting from five configurations indicated in Subsection B. At different configurations, the same motor driving output will cause linkage vibration with different frequency composition. Peaks in Fig. 10 are of different frequencies and different amplitudes. It is clearly shown that the influence of rigid body motion on flexible motion is configuration dependent. When a command is sent to a
motor, overall rigid body motion of the parallel platform is involved because of the coupling of rigid body motion and flexible motion, and the closed loop kinematic structure. In order to efficiently suppress the linkage vibration, the configuration dependency must be considered.

D Discussion of configuration dependency

From the analyses performed, it is observed that configuration-insensitive linkage mode shapes are included in mode shapes extracted from the configuration dependent transfer function from motor voltage input to linkage vibration response. In order to evaluate the possibility of control configuration-dependent vibration with linear-time invariant mode shapes, an assumption is made here to separate the transfer functions from motor voltage input to linkage vibration responses \( G(s) \) into the configuration insensitive linkage dynamics \( H(s) \) and the configuration dependent part \( O(s) \):

\[
G^I(s) = O(s)\{H_1(s) \ H_2(s) \ H_3(s) \ H_4(s)\} \tag{1}
\]

where

\[
H_j(s) = \sum_{i=1}^{c_j} \frac{c_i \phi^{(i)}_j}{s^2 + 2\xi_j\omega_j s + \omega_j^2}, \quad i = 1, 2, 3, 4
\]

\( c_j \) motor-voltage influence factor for the \( j^{th} \) mode,

\( r \) number of modes to keep, \( \phi^{(j)}_i \) the \( j^{th} \) mode shape.

The multiplication of \( O(s) \) and \( d(s) \) will be regarded as the new input to the flexible dynamics. Thus, a configuration dependent vibration control system is transformed to a weakly configuration dependent system through redefining the input. This assumption is the basis for active vibration control design in this paper. By active vibration control, linkage dynamics \( H(s) \) would be modified to suppress the configuration-dependent structural vibration.

IV. VIBRATION CONTROL DESIGN

Taking the assumption made in Subsection III-D, a modal domain vibration control algorithm is designed and experimentally implemented using the parameters identified at the home configuration. A feedback active vibration control scheme is presented and control law design based on the scheme is described. Experimental results are also given.

Fig. 11 shows a schematic of active control system for one flexible linkage in the smart parallel platform. The system is comprised of four components: the transfer function from rigid body motion input to linkage flexible motion \( G(s) \), spatial modal filter \( \{\phi^T \phi\}^{-1} \phi^T \) estimating modal coordinates \( \eta(s) \) from PZT responses \( w(s) = \{w_1(s) \ w_2(s) \ w_3(s) \ w_4(s)\}^T \), modal domain controller \( C(s) = \{C_1(s) \ C_2(s)\} \) with PZT output \( u(s) = \{u_1(s) \ u_2(s)\} \), PZT actuator-to-linkage vibration transfer function matrix \( P(s) = \{P_1(s) \ P_2(s)\} \),

\[
P_i(s) = \sum_{j=1}^{3} \frac{p_{ij} \phi^{(j)}_i}{s^2 + 2\xi_i \omega_i s + \omega_i^2}, \quad i = 1, 2, \text{ where } p_{ij} \text{ the } j^{th} \text{ modal influence factor for the } i^{th} \text{ actuator.}
\]

In Section III, \( G(s) \) has been discussed thoroughly and \( P(s) \) has been identified. The vibration controller in the modal domain to suppress structural vibration is based on spatial modal filters. In the system, the bandwidth of motor drive signal \( d(s) \) is determined to be 250Hz. Hence, only the first mode of the linkage is targeted here. However, cautions must be taken to avoid exciting the second mode, which might lead to instability, i.e. so-called spillover.

Use of the pseudo-inverse of identified mode shapes, i.e. \( \{\phi^T \phi\}^{-1} \phi^T \), as spatial modal filters to extract modal coordinates has been proposed by Morgan [17], when there are more vibration sensors than the number of modes. This condition is also called the spatial sampling theorem. However, vibration of the flexible linkages has many frequency components other than two natural frequencies. Attempts to use two mode shapes in Fig. 7 as modal filters shows a spatial filter maintains vibration around that natural frequency and attenuates vibration far from the natural frequency.

The vibration response of the linkage is the superposition of elastodynamics response caused by rigid body motion and the active vibration control response:

\[
w(s) = G(s)d(s) + P_1(s)u_1(s) + P_2(s)u_2(s) \tag{2}
\]

The transfer function from motor \( l \) input to PZT sensor \( i \) response is:

\[
w_i(s) = \frac{O(s) \cdot \frac{c_i \phi^{(i)}_l}{s^2 + 2\xi_i \omega_i s + \omega_i^2}}{1 - \sum_{k=1}^{3} \frac{p_{ik} \phi^{(k)}_i}{s^2 + 2\xi_k \omega_k s + \omega_k^2}} \tag{3}
\]

Equation (3) is rewritten as,

\[
w_i(s) = \frac{O(s)c_i \phi^{(i)}_l}{s^2 + 2\xi_i \omega_i s + \omega_i^2 - \sum_{k=1}^{3} C_i(s) p_{ki} \phi^{(k)}_i} \tag{4}
\]
The vibration controller $\mathbf{C}(s)$ is designed based on the spatial modal filter,
\[
\mathbf{C}(s) = \begin{bmatrix} C_1(s) \\ C_2(s) \end{bmatrix} = \begin{bmatrix} K_{H1} + K_{D1}s \\ K_{H2} + K_{D2}s \end{bmatrix}
\]

(5)

$\mathbf{C}(s)$ changes the sensitivity of the linkage vibration over the motor voltage input by modifying the closed-loop poles of the linkage dynamics, i.e. the roots of the denominator of $w(t)/d(t)$. Computer simulation has been undertaken to determine the feedback control gain using the Eigenvalue Assignment [1] method which places the system poles on the complex plane. A constraint is set to prevent the control output from exciting the second mode of the linkage.

Control gains are solved as:
\[
K_{H1} = -0.15, K_{D1} = -0.39
\]
\[
K_{H2} = -0.249, K_{D2} = -0.6474
\]

(6)

Experiments have been conducted to verify the effectiveness of this control design. Linear spectrums of four PZT sensors show that vibration amplitude with the frequency range of 60–200 Hz is decreased by 40%–50% at 147 Hz. Fig. 12 shows the linear spectrum of PZT sensor 2, which has the highest vibration amplitude. Amplitude at 108 Hz is decreased by 51% and 53.3% at 147 Hz. Vibration outside of this frequency range is slightly increased but will not compromise the control effect within the frequency range.

VI. CONCLUSIONS

In this paper, modal analysis experiments using an impact hammer, an accelerometer, and PZT transducers are performed. While displacement mode shapes of flexible linkage using hammer-accelerometers test, found to be consistent with those using PZT transducers, is determined weakly configuration dependent, the influence of the rigid body motion on linkage flexible motion, i.e. the vibration characteristic of parallel platform, is determined strongly configuration dependent. An assumption is made that the influence of rigid body motion on flexible motion is separated into a configuration dependent part and a configuration independent part so constant mode shapes are able to be used in the vibration control design. Based on this conclusion, a modal control strategy is designed using modal parameter at the home configuration. Active vibration experimental results show a 40%-50% reduction of vibration amplitudes. Vibration control of configuration-dependent modal parameters is currently under investigation.

REFERENCES